precipitation on trees, &c., which is increased by wind, and large drops of rain on to the ground beneath them. This condition seems best explained by Aitken's discoveries showing the possibility of a super-saturation of air when the number of dust-particles is unusually small in a mass of air which is humid and cooled to saturation. The dust-particles from their minuteness, and from their inability to fall below the temperature of the air owing to the cloud canopy above, do not condense much of the vapour, and consequently any solid object of the same or slightly lower temperature brings about precipitation from the passing air, which may possibly be super-saturated. A slight fall of temperature in the air, or sometimes an increase of dust-particles, then produces fog. A dry fog may thus result from cold causing condensation on a very large number of dust-particles which are radiating heat rather freely, and a damp mist from partial condensation from super-saturated air on a comparatively small number of dust particles not radiating freely owing to a clouded sky.

These considerations explain why a dry fog is densest in London and a wet mist densest in the country. A dry fog is the work of cold radiating particles, a wet mist is the work of cold air mixing with warm. "In a fog," says Angus Rankin,[1] "the watery vapour in condensing has more particles to condense on, and consequently the particles of fog are smaller, and on meeting with an object with a higher temperature, instead of wetting it the object dries them up by parting with some of its heat. On the other hand, in a mist, the particles of dust, being few, have more water condensed on each, and so are larger and do not readily evaporate with small increments of heat." Yet in a damp mist the addition of a large number of dust-particles, as in a town by day, scarcely increases the density of the mist. In fact, the wet mist is less dense in London than in the country, owing to the higher temperature and lower humidity of the air. Dry or radiation fogs, which cling to the ground, are the most dense in smoky places.

In fogs with frost in winter, such as have occurred several times in the last few years, I have always found the windward side of objects to be much more heavily frosted than the leeward, and the rime to attach itself most to points and edges. Trees have thus become laden with rime, even so as to break down branches; iron points of railings, splinters of wood, wires, and blades of grass have borne spikes and fern-like growths an inch or more long, and heather and fern in hollows have been whitened as if with a fall of snow. In weather of this kind it is difficult to say what is dew or frost proper, and what is deposited moisture from super-saturated air and from fog. On the same night a white frost may present the characteristics of fog-deposition in a valley and of clear condensation on a neighbouring hill.

Dew and frost are in fine the result of many causes which inter-operate in a complex manner. The importance of the laws of gases of the multitude of fine adaptations in the relations of vapour, air, water, earth, and plants; the importance, too, of the thermal receptivity of boundless space, gives an interest to this branch of meteorology which is second only to its beauty.

ARBORESCENT FROST PATTERNS.

PROF. MELDOLA'S account of Arborescent Frost Patterns has excited a good deal of interest, and we have received many letters on the subject, some of which we have already published. To-day we give reproductions of photographs we have received from Mr. J. Maclear, Cranleigh. Fig. 1 represents a photograph of a facsimile tracing of a "Nature print" of an ice crystallite taken by Mr. A. Anderson on a still and sunny early morning in January 1887, after a not very severe frost. The sunshine had dried the rest of the frost off the flagstone, and left this mud and ice-crystallization, which he promptly secured on soft paper by means of a soft pad-pressure, and thus got a perfec Nature printed impression. The original (now unfortunately lost) showed an appearance of vegetable (moss) growth, even more strikingly than in this tracing from it.

With regard to Fig. 2 Mr. Maclear writes:—"The melting ice under the dabbing pad formed a natural pigment with the mud on the flagstone, the rest of the flagstones being perfectly dry already by the early morning sunshine."

Prof. Meldola sends us the following interesting letter which he has received from Corbridge-on-Tyne:—

"I was much interested by your note in NATURE the other day, anent the frost markings of a vegetable pattern. I have seen just the same forms several times in the north, but it is I think the least common of the patterns usually met with. I write, however, to call your attention to Figs. 1 and 7 of Plate vii, illustrating the article on Meteorology in the "Encyclopædia Metropolitana" (1845, vol. i. of plates, vol. v. of text). These figures are very like yours and some of the others given with them are also very interesting. I have often shown my students when out in the fields in cold weather how exactly the mud-cum-frost markings of the common feathery volute type imitate the so-called *Cauda-galli* fossil fucoid (?) which is one of the most abundant objects on the surface of the carboniferous limestone courses about here. As far as form goes they are identical, and there is no structure to be discovered in the fossil markings.

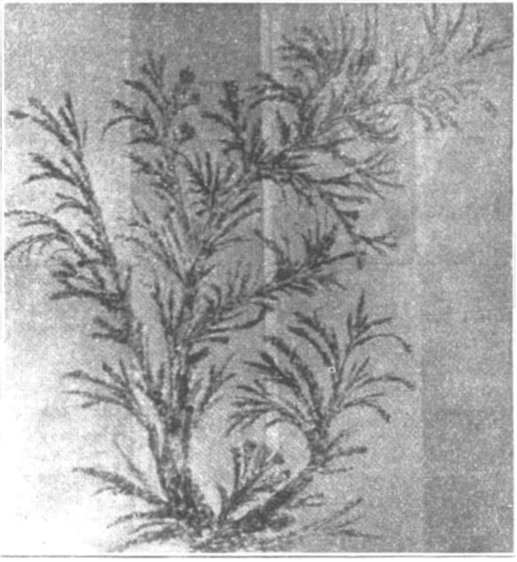

FIG. 1.—Ice crystallite, "Nature printed" by A Anderson, January 1887. Facsimile tracing by J. Maclear, January 1887. Size of crystal 14¾ inches × 13⅜ inches.

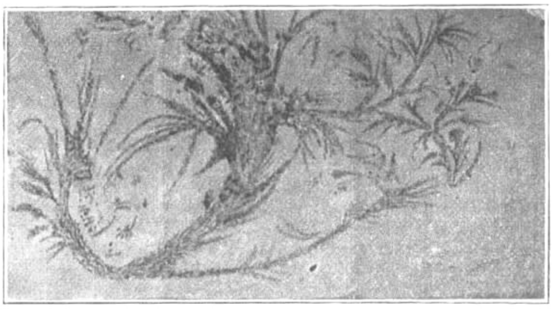

FIG 2.—Photograph from the original "Nature print," made Mr. A Anderson, of an ice crystallite, January 1887. Size of crystal, 10¾ inches × 7 inches.

"Corbridge-on-Tyne, December 16. G. A. LEBOUR."

PROF. SOLLAS writes to us:—The correspondence on this subject that has lately appeared in your columns (particularly Prof. Bonney's reference), leads me to anticipate a communication I hope shortly to present to the Royal Dublin Society on the growth of crystals. The arborescent forms assumed by ice are merely a special case of a very general problem—that of the

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[1] *Journal of the Scottish Meteorological Society.* Third Series. No. viii

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forms assumed by crystals under different conditions. Petrologists have long been familiar with the tendency of crystals developing in a viscid medium to excessive growth in one or more directions. Felspar is a familiar instance, the lath-like forms which it frequently assumes being due to elongation along one axis (x), the length of prisms measured along this axis often exceeding by ten times that along the axis y or z. The cause of this need not now be discussed; it will be sufficient to add that the phenomenon is not special to felspar, but is of quite general occurrence. With this tendency is connected the origin of curvilinear forms. We may consider the molecules forming the growing face of a long prism; the spheres of influence of these lie half within and half without the substance of the crystal. Considering this influence as attractive (directly or indirectly), we may say that the attraction of the molecules leading to further deposition is one-half their total attraction. If now from the face we pass to the edge between two faces at right angles, only one-quarter of the sphere will be immersed, and the attraction may be spoken of as three-quarters of the whole; while if from the edge we pass to a corner, only one-eighth is immersed, and the attraction becomes seven-eighths. From this it follows that growth should be more rapid at the edges than over the surface of the face, and still more rapid at the corners. In accordance with this we find young growing prisms in a viscid medium increasing so rapidly at the edges as to leave a space in and about the axis filled with the medium in a non-crystalline state. Indeed, a viscid medium is not necessary; hollow prisms are of common occurrence whenever crystallization takes place with rapidity. Further, in quite embryonic crystallites, Vogelsang figures elongated prism-like forms, in which the four corners are produced parallel to the long axis into processes resembling spines. There is an additional reason pointed out to me by Prof. Fitzgerald why growth should be more rapid at the edges and corners than over the general surface, and that is that these parts are more exposed to molecular bombardment.

If crystals are more readily built up along edges and corners, we should expect them to be more readily unbuilt in these regions, and this is in accordance with observation; the zonal felspars of igneous rocks, in the formation of which intervals of solution have alternated with periods of growth, usually present, in the outlines of each resulting envelope, rounded corners.

The influence of corners is well seen in some glassy rocks where small prisms of felspar (andesite) may be observed, with five or six slenderer but longer prisms springing from a corner in radiate divergence.

From this it is but a step to curvilinear growth. Let a prism tend to rapid rectilinear growth, and any check immediately in front will lead to a forward growth from a corner in a slightly different direction; even the competition of molecules for this centre of attraction may by overcrowding bring about this result, and thus both branching and curvilinear forms may arise. This is beautifully exemplified in the spherulites of many igneous rocks, where we find in the centre of a radiately crystallized sphere a long prism of felspar serving as a nucleus, and from the *ends* of this slender, almost linear, prisms diverge towards a spherical surface which by repeated branching and associated curving they everywhere reach, leaving about the sides of the nucleus a spherical space almost devoid of crystal structure. The whole arrangement in median longitudinal section presents a remarkable resemblance to the lines of force as shown by iron-filings about a bar magnet.

Evidently in rapid crystallization with a tendency to linear growth, divergence may be repeated at such frequent intervals as to produce forms which to the unaided eye appear to be continuous curves.

SOCIETIES AND ACADEMIES.

London.

Mathematical Society, December 8.—Mr. A. B. Kempe, F.R.S., President, in the chair.—The following communications were made:—On a theorem in differentiation, and its application to spherical harmonics, by Dr. Hobson.—On Cauchy's condensation test for the convergency of series, by Prof M. J. M. Hill. Cauchy's condensation test for the convergency of series is as follows:—If $f(n)$ be positive for all values of n, and constantly decrease as n increases, then the series $\Sigma f(n)$ and $\Sigma a^n f(a^n)$ are both convergent or both divergent, where a is any positive integer not less than 2. There is a clear reason why a cannot be unity, for then $\Sigma a^n f(a^n) = \Sigma f(1)$, which is always infinite. It is proved in Chrystal's "Algebra" that the theorem is also true if a have any positive fractional value not less than 2, see part 2, chapter xxvi., § 6, cor. 1. The proof there given when a lies between the consecutive positive integers p and $p + 1$ is based on Cauchy's proof for the two cases $a = p$ and $a = p + 1$. But this proof will not apply when $1 < a < 2$, because Cauchy's proof will not apply when $a = 1$. Yet it does not seem possible to assign a reason for excluding values of a between 1 and 2, for Cauchy's method appears to depend on this—viz. that for increasing values of n the expression $f(a^n)$ occupies more and more advanced positions amongst the terms of the series $\Sigma f(n)$; but this is possible if $1 < a < 2$, as well as when $a > 2$. If $a \leqq 1$, then this is no longer true. The problem then considered in this paper is so to recast the proof for fractional values of a as not to exclude the case $1 < a < 2$. The complete theorem will then stand thus:—If $f(n)$ be positive for all values of n, and constantly decrease as n increases, then the series $\Sigma f(n)$ and $\Sigma a^n f(a^n)$ are both convergent or both divergent if $a > 1$. The demonstration depends on the following theorems.

I. If $\Sigma a^n f(a^n)$ be convergent, then—

$$\sum_{n=1}^{n=\infty} f(n) < f(1) + \ . \ . \ . \ + f(t_s)$$

$$+ (a - 1 + a^{-s}) \left[\sum_{n=1}^{n=\infty} a^n f(a^n) - \sum_{n=1}^{n=s-1} a^n f(a^n) \right]$$

where s is any integer so large that

$$a^{s+1} - a^s > 1,$$

and t_s is the greatest integer in a^s, a being greater than 1.

II (A). If $\Sigma a^n f(a^n)$ be divergent, and if $a^n f(a^n)$ diminish as n increases beyond a certain value, then

$$\sum_{n=1}^{n=\infty} f(n) > f(1) + \ . \ . \ . \ + f(t_s)$$

$$+ (1 - a^{-1} - a^{-s-1}) \left[\sum_{n=1}^{n=\infty} a^n f(a^n) - \sum_{n=1}^{n=s} a^n f(a^n) \right]$$

II (B.) If $\Sigma a^n f(a^n)$ be divergent, and if $a^n f(a^n)$ do not diminish as n increases beyond a certain value, then—

$$\sum_{n=1}^{n=\infty} f(n) > \sum_{n=1}^{n=s} f(n) + \sum_{n=s+1}^{n=\infty} \mathrm{A} n^{-1},$$

where s is an integer taken large enough, and A is some finite quantity.—Additional note on secondary Tucker circles, by Mr. J. Griffiths.—Notes on determinants, by Mr. J. E. Campbell. In accordance with the late Prof Smith's notation, a determinant of the p^{th} class may be written

$$| a_{ijk} \ . \ . \ . \ |$$

The fact that a determinant of the second class (an ordinary determinant) is not altered if the vertical columns be written horizontally is expressed by the identity

$$| a_{ij} | = | a_{ji} |$$

For determinants of higher class it is known that any of the suffixes can be interchanged, except the first: and if the class be even, the first suffix can also be interchanged with any other, but for determinants of odd class this is not true. By considering a cubic determinant as an ordinary determinant in alternate numbers, the author tries to explain this essential distinction between determinants of odd and even classes. If the element $a_{pqr \ . \ . \ .} = - a_{qpr \ . \ . \ .}$, and $a_{ppr} = 0$, the determinant is called skew symmetrical. It is easily seen that skew symmetrical determinants of even class and odd degree vanish identically. This is analogous to the well-known theorem in ordinary determinants; but there is no corresponding analogue to the theorem that skew symmetrical determinants of the second class and even

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